

A modified “bottom-up” thermalization in heavy-ion collisions

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Abstract. In the initial stage of the bottom-up picture of thermalization in heavy-ion collisions, the gluon distribution is highly anisotropic which can give rise to plasma instability. This has not been taken into account in our original paper (Phys. Lett. B **632**, 257 (2006) hep-ph/0505164). It is shown that in the presence of instability there are scaling solutions, which depend on one parameter, that match smoothly onto the late stage of bottom-up when thermalization takes place.

PACS. 25.75.-q Relativistic heavy-ion collisions – 12.38.Mh Quark-gluon plasma – 52.27.Ny Relativistic plasmas – 52.35.Qz Microinstabilities (ion-acoustic, two-stream, loss-cone, beam-plasma, drift, ion- or electron-cyclotron, etc.)

1 The original bottom-up picture

In the McLerran-Venugopalan model of the color glass condensate [1], small- x gluons with transverse momentum below a certain saturation scale Q_s are at their maximum density. When applied to a nucleus-nucleus collision at impact parameter b , this scale is given by [2]

$$Q_s^2 = \frac{8\pi^2\alpha N_c}{N_c^2 - 1} \sqrt{R_A^2 - b^2} \rho x G_p(x, Q_s^2) \quad (1)$$

and its value is $Q_s \sim 1$ GeV at the relativistic heavy-ion collider (RHIC). Here R_A is the nuclear radius, ρ is the nuclear number density, N_c is the number of color, α is the coupling and G_p is the gluon distribution of a proton. In a nuclear collision these gluons have a typical momentum of Q_s and are freed at a time around $1/Q_s$ after the initial impact. In the bottom-up picture, which is based on the observation that inelastic processes are no less important than elastic processes for thermalization [3], equilibration is driven by these hard gluons and it goes through three distinct stages [4]. They are a) the early times $1 < Q_s\tau < \alpha^{-3/2}$, b) the intermediate times $\alpha^{-3/2} < Q_s\tau < \alpha^{-5/2}$ and c) the final stage $\alpha^{-5/2} < Q_s\tau < \alpha^{-13/5}$.

1.1 a) $1 < Q_s\tau < \alpha^{-3/2}$

At the early times hard gluons dominate and because of the longitudinal expansion the density goes down like

$$N_h \sim \frac{Q_s^3}{\alpha(Q_s\tau)}. \quad (2)$$

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In the central collision region most of the gluons have small longitudinal momentum, $p_z \ll 1$, otherwise they would have wandered out of the region. But this momentum cannot be zero either because of broadening due to multiple scattering. Effectively, the p_z goes through a random walk in momentum space due to the random kicks by other hard gluons so

$$p_z^2 \sim N_{\text{col}} m_D^2 \sim \frac{\alpha N_h}{p_z}, \quad (3)$$

where N_{col} is the number of collisions a hard gluon typically has encountered at the time τ and m_D^2 is the screening mass square

$$m_D^2 \sim \alpha \int d^3p \frac{f_h(p)}{p} \sim \frac{\alpha N_h}{Q_s} \sim \frac{Q_s^2}{Q_s\tau} \quad (4)$$

which effectively acts as the variance for each kick due to the much more frequent small-angle collisions. p_z comes out to be

$$p_z \sim (\alpha N_h)^{1/3} \sim \frac{Q_s}{(Q_s\tau)^{1/3}}. \quad (5)$$

Soft gluons with momentum k_s are produced during these times via the Bethe-Heitler formula [5] to give the parametric form for N_s

$$N_s \sim \tau \frac{\partial N_s}{\partial \tau} \sim \frac{Q_s^3}{\alpha(Q_s\tau)^{4/3}}. \quad (6)$$

Once produced, random scattering by other gluons energizes these soft gluons so that their momenta settle around $k_s \sim p_z$. Therefore the soft-gluon distribution becomes

$$f_s \sim \frac{N_s}{k_s^3} \sim \frac{1}{\alpha(Q_s\tau)^{1/3}}. \quad (7)$$

1.2 b) $\alpha^{-3/2} < Q_s \tau < \alpha^{-5/2}$

In the intermediate times hard gluons still dominate in numbers but now $f_h < 1$. This changes the scattering rate with the hard gluons so

$$k_s^2 \sim N_{\text{col}} m_D^2 \sim \alpha Q_s^2 \quad (8)$$

is now a constant. Assuming that the screening is mainly due to the soft gluons

$$m_D^2 \sim \frac{\alpha N_s}{k_s} \gg \frac{\alpha N_h}{Q_s}, \quad (9)$$

one can find self-consistently that

$$N_s \sim \frac{\alpha^{1/4} Q_s^3}{(Q_s \tau)^{1/2}}. \quad (10)$$

1.3 c) $\alpha^{-5/2} < Q_s \tau < \alpha^{-13/5}$

In the final stage most gluons are soft $N_s \gg N_h$. The remaining hard gluons will scatter with the soft gluons and lose energy via successive gluon splitting. Whereas in the previous stages gluon production via the Bethe-Heitler formula is unaffected by multiple scattering, this is no longer true as the branching gluon momenta now fall within the range of the Landau-Pomeranchuk-Migdal suppression [6]. Specifically gluon emission with momentum larger than $k_{\text{LPM}} = m_D^2 / N_{\text{scatt}} \sigma$ is suppressed [7]. N_{scatt} is the number density of the particles that is responsible for most of the scatterings. In this case the formation time of the branching gluon is $t_f \sim k_{\text{br}} / k_t^2$ where k_t is the transverse momentum picked up by the branching gluon through the random kicks by the soft gluons. It can be estimated as momentum broadening as before but the number of collisions is now restricted by the formation time t_f and the mean free path λ , hence

$$k_t^2 \sim m_D^2 t_f / \lambda. \quad (11)$$

The rate of branching is roughly related to the formation time via $1/t_{\text{br}} \sim \alpha/t_f$. Equating t_{br} with τ and requiring that the soft gluon now be in a thermal bath $N_s \sim T^3$, one finds the branching momentum to be

$$k_{\text{br}} \sim \alpha^4 T^3 \tau^2. \quad (12)$$

Lastly, equating the energy flow from the hard gluons to the soft thermal bath, the temperature is determined to have the linear time dependence

$$T \sim \alpha^3 Q_s^2 \tau. \quad (13)$$

We will see later on how some of these parametric dependences are recovered even after instability is included into the consideration.

2 The instability

As mentioned previously, early on in the collision only small- x gluons can remain in the central region and they

have typical transverse momentum of the order of Q_s . This describes a picture of gluons with highly anisotropic initial momentum distribution. In such a situation as pointed out a long time ago [8] and more recently within the context of the bottom-up picture [9], it would give rise to plasma instability. The instability occurs because the dispersion relation for the soft gluons gives a negative value for the screening mass square:

$$m_D^2 \sim -\frac{\alpha N_h}{Q_s} \quad (14)$$

when the momentum distribution is highly anisotropic [10]. Modes with momentum $k < m_D$ are unstable. For recent reviews on the topic of instability in the context of heavy-ion collisions, one can read for example [11]. Although the growth is exponential in nature¹ and should be very fast on the time scale of $\tau \sim 1/m_D$ or $Q_s \tau \sim 1$, it is difficult for it to lead directly to equilibration because, first, the instability only produces soft particles and, second, Arnold and Lenaghan (the first paper of [10]) showed that equilibration cannot occur before $Q_s \tau \sim \alpha^{-7/2}$ which is much later than $Q_s \tau \sim 1$ for small α_s .

Instability creates many soft gluons as a result. There are two possibilities for the system to evolve further:

- i) When the soft particles are saturated at $f_s \sim 1/\alpha$ further production via the instability will result in gluons with $k \sim m_D$ being transferred to higher momenta.
- ii) Or the instability will be completely eliminated by the soft gluons at saturation.

In either case, in the same spirit of the bottom-up picture, it is natural to look for a scaling solution which connects the end of the exponential growth due to the instability to final equilibration.

3 A possible scaling solution

The solution(s) that we propose of course still has to start with the longitudinally expanding initial hard gluons

$$N_h \sim \frac{Q_s^3}{\alpha(Q_s \tau)}. \quad (15)$$

For gluons produced sometimes after the beginning but before τ , $1/Q_s < \tau_0 < \tau$, these have [14]

$$\begin{aligned} N_s(\tau, \tau_0) &\sim \frac{Q_s^3}{\alpha(Q_s \tau)(Q_s \tau_0)^{1/3-\delta}}, \\ k_s(\tau_0) &\sim \frac{Q_s}{(Q_s \tau_0)^{1/3-2\delta/5}}, \\ \alpha f_s(\tau, \tau_0) &\sim \frac{(Q_s \tau_0)^{1/3+\delta/5}}{(Q_s \tau)^{2/3+2\delta/5}}, \end{aligned} \quad (16)$$

¹ In [12] it was shown that at late times the growth changed character from an exponential to a linear one and in [13] for an longitudinally expanding plasma, the exponent was shown to be $\sim \sqrt{\tau}$ as one would expect from the form of eq. (4).

where $N_s(\tau, \tau_0)$ is the number density of particle produced at time τ_0 but measured at τ .

For gluons produced at time τ , one can write down a family of δ -parameter-dependent scaling solutions [14]

$$\begin{aligned} N_s &\sim \frac{Q_s^3}{\alpha(Q_s\tau)^{4/3-\delta}}, & k_s &\sim \frac{Q_s}{(Q_s\tau)^{1/3-2\delta/5}}, \\ \alpha f_s &\sim \frac{1}{(Q_s\tau)^{1/3+\delta/5}}, & m_D &\sim \frac{Q_s}{(Q_s\tau)^{1/2-3\delta/10}}, \end{aligned} \quad (17)$$

where $\delta \geq 0$. At $\delta = 0$ they coincide with the initial parametric form of the original bottom-up picture described in the first section. The solutions obey

$$m_D^2 \sim \frac{\alpha N_s}{k_s}, \quad (18)$$

$$N_s \sim \tau \frac{\alpha^3}{m_D^2} (N_s f_s)^2, \quad (19)$$

$$k_s^2 \sim m_D^2 \frac{\tau}{\tau_{\text{col}}} \quad \text{with} \quad \frac{1}{\tau_{\text{col}}} \sim \frac{\alpha^2}{m_D^2} N_s f_s, \quad (20)$$

$$\frac{1}{\tau} \sim \frac{\alpha^2}{k_s^2} N_s f_s. \quad (21)$$

Here m_D at τ is determined by soft gluons produced via the Bethe-Heitler formula in eq. (19). Multiple scattering ensures that these gluons gain momentum until they reach a value around k_s given by eq. (20). Once there, they scatter once on the average so they are borderline as far as reaching equilibrium.

4 The value of δ and $m_D^2 > 0$?

So far we have always given the mass m_D a subscript of D which stands for the Debye screening mass but in all reality, we are uncertain about the sign of the mass square. In sect. 2 we pointed out that the initial momentum distribution was highly anisotropic, thus some soft gluon modes were unstable. Looking at the problem only parametrically as done in the bottom-up picture and also here would not help us ascertain the sign of m_D^2 . More dynamical inputs are necessary. One can compare the momentum distribution and from the degree of anisotropy deduce whether m_D^2 is negative. But the problem is more complicated than that. For example, from eq. (16) the contribution of the gluons produced at τ_0 to the screening mass square is

$$m_D^2(\tau, \tau_0) \sim \frac{\alpha N_s(\tau, \tau_0)}{k_s(\tau_0)} \sim \frac{Q_s^2(Q_s\tau_0)^{3\delta/5}}{Q_s\tau}. \quad (22)$$

If $\tau_0 \ll \tau$ then this contribution is clearly negative because $k_s(\tau_0)$ is so dissimilar to $k_s(\tau)$. However, the contribution is small compared to m_D^2 , which as seen in eq. (17), has the same expression as eq. (22) except τ_0 is τ in this case. On the other hand, if $\tau_0 \sim \tau$ then the gluon’s momentum distribution tends to be isotropic and it is unlikely that m_D^2 is negative. The more difficult case is $\tau_0 < \tau$ when one can no longer be certain when the sizes of the contributions to

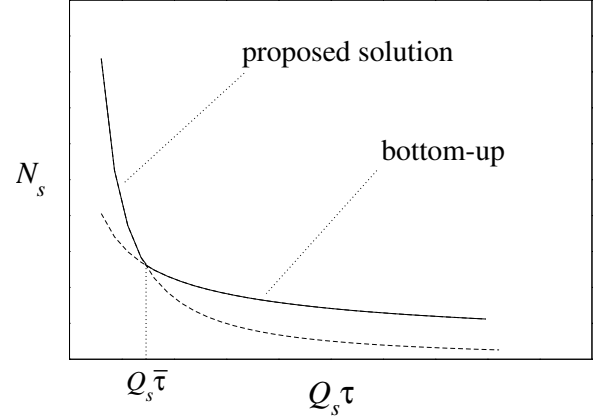


Fig. 1. Graphical representation of the matching of the scaling solution to the bottom-up picture at $Q_s\bar{\tau}$.

m_D^2 are comparable. It is here that the parameter δ plays a role since the ratio of the late gluon to the early gluon contribution goes like $(\tau/\tau_0)^{3\delta/5}$. Larger value of δ put more weight on the late-time gluons’ contribution. Better considerations and calculations are necessary to determine the value of δ^2 .

5 Matching onto bottom-up

The solution(s) that we proposed in eq. (17) would not be of any value if it did not describe also the equilibrium phase. In fact at a time when

$$Q_s\bar{\tau} \sim \alpha^{-15/2(5-6\delta)} \quad (23)$$

our scaling solution becomes identical to the intermediate stage, $\alpha^{-3/2} < Q_s\tau < \alpha^{-5/2}$, of the bottom-up picture when the basic quantities in both cases go like

$$\begin{aligned} N_s &\sim Q_s^3 \alpha^{\frac{10-3\delta}{2(5-6\delta)}}, & k_s &\sim Q_s \alpha^{1/2}, \\ f_s &\sim \alpha^{\frac{5(-1+3\delta)}{2(5-6\delta)}}, & m_D &\sim Q_s \alpha^{\frac{3(5-3\delta)}{4(5-6\delta)}}. \end{aligned} \quad (24)$$

This is true provided $0 < \delta < 1/3$. A graphical representation of this is shown in fig. 1. At this time the present solution should make a transition into the original bottom-up solution which remains true for the rest of the evolution as long as the intermediate stage of the bottom-up picture is not too affected by the initial presence of the instability.

For the case when $\delta > 1/3$, we can see from eq. (24) that f_s approaches unity. In fact in that case at a time $Q_s\tau_1 \sim \alpha^{-15/(5+3\delta)}$ already $f_s \sim 1$. Much of the picture of the final stage of the bottom-up becomes true except that gluons produced early at time τ_0 now play the part

² Bödeker considered the broadening of the p_z by multiple scattering with the much denser unstable gluon modes instead of with the hard gluons [15]. In that case he found $p_z \sim Q_s/(Q_s\tau)^{1/4}$, which would suggest a value for $\delta \sim 5/24 < 1/3$ provided that p_z takes this parametric form until the moment when the instability was finally eliminated.

of the hard particles since $N_s(\tau, \tau_0) > N_h$ and $k_s(\tau_0)$ now functions as the branching momentum k_{br} in eq. (12)

$$k_s(\tau_0) \sim \alpha^4 T^3 \tau^2. \quad (25)$$

The transfer of energy is similarly via gluon branching from these gluons into the bath of soft gluons. Equating once again the energy flow from these gluons into the thermal bath with temperature T ,

$$\frac{d\epsilon}{d\tau} \sim T^3 \frac{dT}{d\tau} \sim \frac{N_s(\tau, \tau_0)}{\tau} k_s(\tau_0), \quad (26)$$

using eq. (16) and eq. (25) in eq. (26), one finds

$$T \sim Q_s \alpha^{\frac{35-78\delta}{39\delta-10}} (Q_s \tau)^{\frac{15-36\delta}{39\delta-10}}. \quad (27)$$

At $\delta = 1/3$, this takes the familiar form $T \sim \alpha^2 Q_s^2 \tau$ of a linear increase of T with τ which is characteristic of the bottom-up picture in [4]. This heating up of the bath of soft gluons ends when the transfer of energy to the thermal bath is complete. This occurs when the branching momentum $k_s(\tau_0)$ in eq. (25) finally reaches Q_s and

$$T^4 \sim N_h(\tau). \quad (28)$$

At this time $Q_s \tau \sim \alpha^{-13/5}$. Substituting this into eq. (27), one gets

$$T \sim Q_s \alpha^{2/5}, \quad (29)$$

a value that is independent of δ . One sees that independent of what value δ takes, as long as $\delta > 1/3$, the scaling solutions match up to the final stage of bottom-up only at the final time $Q_s \tau \sim \alpha^{-13/5}$.

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